Course Code: 1960327 **Roll No:** **MLRS-R19**

MARRI LAXMAN REDDY
INSTITUTE OF TECHNOLOGY AND MANAGEMENT

(AN AUTONOMOUS INSTITUTION)
(Approved by AICTE, New Delhi & Affiliated to JNTUH, Hyderabad)
Accredited by NBA and NAAC with 'A' Grade & Recognized Under Section2(f) & 12(B)of the UGC act,1956

III B.Tech II Sem Regular End Examination, June 2022
Finite Element Methods
(Mechanical Engineering)

Time: 3 Hours. **Max. Marks: 70**

Note: 1. Question paper consists: Part-A and Part-B.
2. In Part – A, answer all questions which carries 20 marks.
3. In Part – B, answer any one question from each unit.
Each question carries 10 marks and may have a, b as sub questions.

PART- A

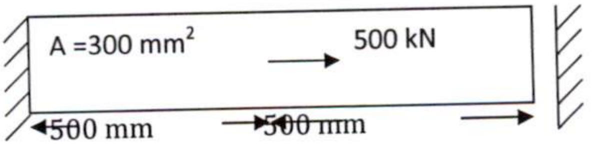
(10*2 Marks = 20 Marks)

1.	a)	What are the important applications of finite element methods?	2M	CO1	BL2
	b)	State the stress – displacement relations.	2M	CO1	BL1
	c)	Write the Stiffness matrix for two noded truss element.	2M	CO2	BL2
	d)	Explain the significance of Hermite shape functions	2M	CO2	BL3
	e)	Write the plain stress functions for three noded triangular element	2M	CO3	BL3
	f)	What is the strain displacement relation matrix for axi-symmetric element?	2M	CO3	BL2
	g)	Write the similarities between conduction and convection.	2M	CO4	BL4
	h)	How convection is taken into account finite element formulation?	2M	CO4	BL3
	i)	Write mass matrix for the stepped bar element.	2M	CO5	BL2
	j)	Discuss about ANSYS software.	2M	CO5	BL3

PART- B

(10*5 Marks = 50 Marks)

2	a)	What is 1-D Linear element? How it is being analyzed in finite element analysis?	5M	CO1	BL2
	b)	Explain the general procedure of finite element analysis step by step with suitable examples.	5M	CO1	BL2
		OR			
3		Estimate the displacement vector, strains, stresses and reactions for following figure . Gap between end of the bar and wall is 2 mm. Take E = 2 X 10^5 N/mm^2	10M	CO1	BL4



4	a)	Explain the Hermite shape functions.	5M	CO2	3
	b)	A beam is fixed at one end and free at the other end, has a 20 kN concentrated load applied at the end of beam having length of 10 m. Find deflection and slope. Take I = 2500 cm^4 and E = 20 x 10^6 N/cm^2.	5M	CO2	3

OR

5 The truss structure shown in figure 3 supports force F at Node 2. FEM is used to analyze the structure using two truss elements as shown: assume $E = 2*10^5 N/mm^2$, $A = 1000\ mm^2$, $L_2=50mm$, $L_1=10mm$ 10M CO2 4

a) Compute the element stiffness matrices for both elements in the global coordinate system and b) Compute the stresses in the elements.

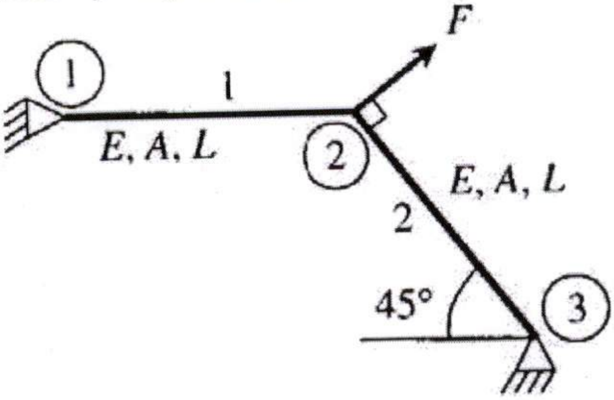


Fig:3

6 a) Evaluate the integral $\int (a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4)\, dx$ with the limits between -1 to +1 using (i) two point Gaussian quadrature and (ii) Analytical integration. 5M CO3 3

b) Derive stress-strain relationship for 2-D CST element. 5M CO3 3

OR

7 Compute the strain displacement matrix and also the strains of a axisymmetric triangular element with the coordinates $r_1 = 3$ cm, $z_1 = 4$ cm, $r_2 = 6$ cm, $z_2 = 5$ cm, $r_3 = 5$ cm, $z_3 = 8$ cm. The nodal displacement values are $u_1 = 0.01$ mm, $w_1 = 0.01$ mm, $u_2 = 0.01$ mm, $w_2 = -0.04$ mm, $u_3 = -0.03$ mm, $w_3 = 0.07$ mm 10M CO3 4

8 a) Estimate the thermal force vector for the 1 D element subjected to end convection and heat flux. 5M CO4 2

b) Derive the finite element equation for 1-D slab. 5M CO4 3

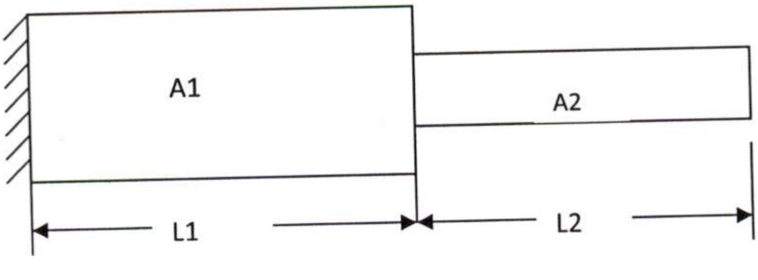
OR

9 Estimate the temperature profile in a pin fin of diameter 30 mm, whose length is 500mm. The thermal conductivity of the fin material is 50 W/m K and heat transfer coefficient over the surface of the fin is 40 W/m² K at 30°C. The tip is insulated and the base is exposed to heat flux of 400 kW/m²K. 10M CO4 4

10 Derive the consistent mass matrix for a 1-D bar element of freedom flexural beam element. 10M CO5 2

OR

11 Find the natural frequencies of longitudinal vibration for a constrained and unconstrained stepped bar as shown in the figure. Where $A_1 = 1000mm^2$, $A_2 = 500mm^2$, $L_1 = 150mm$, $L_2 = 150mm$, assume E= young's modulus, ρ = density 10M CO5 4



EXAMINATION BRANCH

Academic Year	2021-22.
Year & Semester	III & II
Regulation	R-19
Branch	Mechanical
Course Code	1960327
Course Name	Finite Element Method.
Course Faculty's	P. Satya Krishna.
Course Moderator	P. Satya Krishna.
Date of Exam	20/6/22
Reporting Time & Sign	8:30 &

KEY PAPER

QNO	ANSWER	MARKS
1A. a)	PART-A. FEM is extensively used in field of structural mechanics and sucessfly applied to solve other types of Engg: problems → Mechanical → Geo mechanics → civil Engg. → fluid mechanics } structural Heat conduction — For thermal problems	2M.

QNO	ANSWER	MARKS
b)	Strain Displacement relations: $\underset{\text{Strain}}{E} = [B].\underset{\text{Nodal Displacement}}{[q]}$ $E = \frac{dy}{dx} = \frac{d}{dx}\left[N_1 q_1 + N_2 q_2\right]$ $= \left[\frac{dN_1}{dx}, \frac{dN_2}{dx}\right]\begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$ $\epsilon = \left[-\frac{1}{l} \quad \frac{1}{l}\right]\begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$ $B = \frac{1}{l}\left[-1 \quad 1\right].$ $\epsilon = \left[\epsilon_x, \epsilon_y, \epsilon_z, \gamma_{xz}, \gamma_{yx}, \gamma_{xy}\right]^T$	2M
c)	$K = \frac{AE}{l}\begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix}$	2M
d)	Hermite functions are nothing but shape functions in beam.	

QNO	ANSWER	MARKS
	write the formulae is	2M
e)	plane stress functions: plane stress is defined to be a state of stress in which normal stress(σ) and shear stress (τ) directed for to the plane are assumed to be zero. loads act only in x-y plane and members are thin.	2M
f)	write the formulae for strain - displacement matrix formulae.	2M
~~g)~~	~~condition~~ $B = \frac{1}{2A}\begin{bmatrix} \beta_1 & 0 & \beta_2 & 0 & 0 \\ \frac{\alpha_1}{r} + \beta_1 + \gamma_1 z & 0 & \frac{\alpha_2}{r} + \beta_2 + \frac{\gamma_2 z}{r} & 0 & 0 \\ 0 & \gamma_1 & 0 & \gamma_2 & \gamma_3 \\ \gamma_1 & \beta_1 & \gamma_2 & \beta_2 & \beta_3 \end{bmatrix}$	

QNO	ANSWER	MARKS
g.	Conduction: It is amechanism of heat transfer from a region of high temp. to a region of low temperature within a medium (solid, liquid, gas) or different medium in physical contact: Pure conduction is found in solids only. Convection: It is possible in the presence of fluid medium.	2M
h.	Convection is taken has free end convection in slab and outer surfaces in fin.	2M
i.	mass matrix (m) = $\frac{\rho A l}{6}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}$ - consistent $m = \frac{\rho A L}{2}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ — lumped	2M
j	Ansys software: It is at FEM software and write its ~~pro's and cons.~~ Adv. & Dis.	2M

QNO	ANSWER	MARKS

PART-B

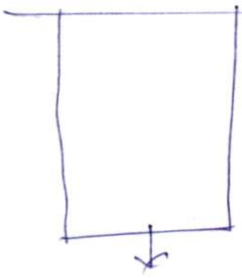
2A)a) 1-D linear Element : A bar and beam elements are considered as 1D elements. The simplest line element is known as linear Element which has 2 Nodes, one at each end.

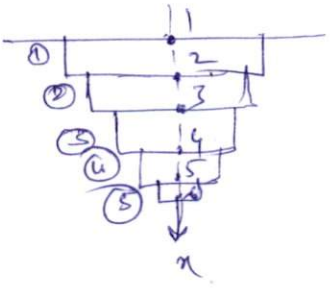
Finite Element analysis consists of

i, Discretization of Structure

ii, Numbering of Nodes.

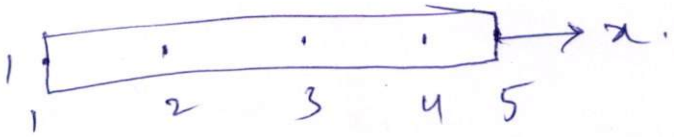
Discretization : It is art of sub-dividing a structure into a convinient number of small components.





QNO	ANSWER	MARKS

Numbering of Nodes : In 1D each node is allowed to move only in ± x direction hence each node as one degree of freedom.



A fix noded Element has five degree of free dom. 5M

Element	Nodes.
①	1 2
②	2 3
③	3 4
④	4 5

connectivity Table.

QNO	ANSWER	MARKS
b)	Step 1: Discretization of Structure: Art of subdividing structure into convinient no. of Smaller Elements. i, 1D ii, 2D iii) 3D iv, Axisymmetric 1D – bar, beam 2D – Triangular, Rectangular 3D – Tetrahedron, Hexahedron Axisymmetric: It is developed by rotating a triangle or quadrilateral about fixed axis. Step-2: Numbering of Nodes & Elements longer side numbering Shorter side numbering in FEM we follow shorter side numbering to	

QNO	ANSWER	MARKS
	reduce memory requirements Step3: Selection of Displacement or Interpolation function. polynomial of linear, quadratic, and cubic form are frequently used as displacement functions. Step4: Define mat. behaviour using strain-displacement and stress-strain relationships $e = \frac{dy}{dx}$. $\sigma = E \cdot e$. Step 5: Derivation of Element Stiffness and Equations. $F = K \cdot U$	5m

QNO	ANSWER	MARKS

Step 6: Assemble the element equations to obtain global eq's:

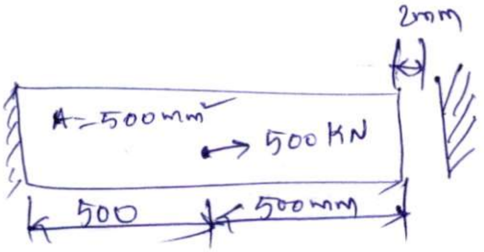
Step 7: Applying BC's

Step 8: Solution for unknown displacement

Step 9: Computation of Element Stresses and Strain.

Step 10: Interpret the results.

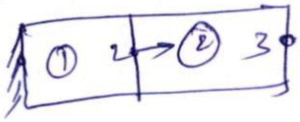
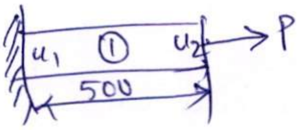
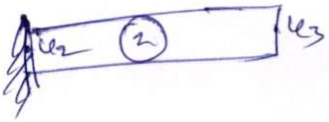
5A)

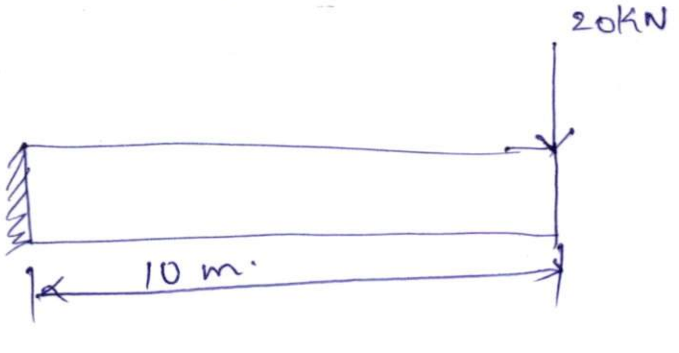


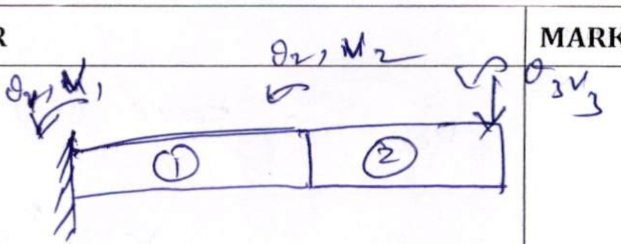
$E = 2 \times 10^5 \, N/mm^2$

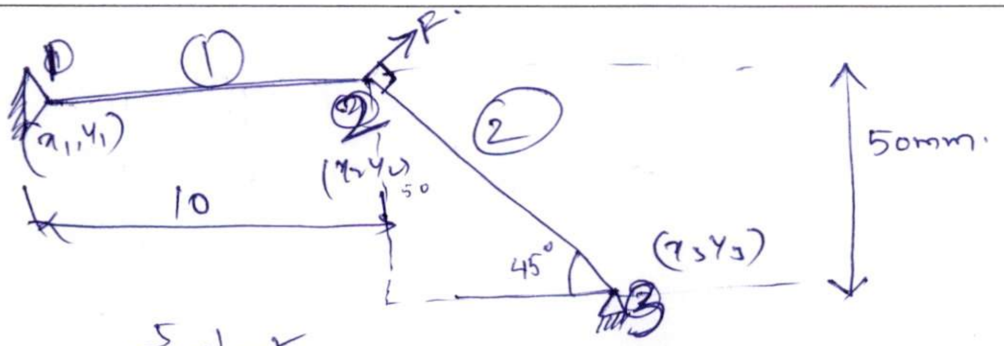
$$\delta L = \frac{PL}{AE} = \frac{500 \times 10^3 \times \overset{\cancel{2}}{5\cancel{0}0}}{\cancel{250} \times \cancel{2} \times 10^5} = 5 \text{ mm.}$$

hence the gap b/w bar and wall is 2mm.

QNO	ANSWER	MARKS
	 Element ① calculate stiffness matrix.  Element ② calculate stiffness matrix  Applying B.C's. $u_1 = 0$, $u_3 = 2mm$, $P = 500 \times 10^3$ N. self wt. is neglected hence $F_1 = F_3 = 0$, $F_2 = 500 \times 10^3 N$ Calculate ~~stress~~ unknow displacement values. calculate stresses $\sigma = E \cdot \frac{du}{dx}$. $\sigma_1 = \frac{E(u_2 - u_1)}{l_1}$, $\sigma_2 = \frac{E(u_3 - u_2)}{l_2}$	6M

QNO	ANSWER	MARKS
	Reaction Force (R) $= R = K.\ u^{0} - F.$ Calculate Reaction forces and is equivalent and opposite to applied force.	4M.
4A)	a) Explain Hermite shape functions write the formulae and derive the shape functions $N_1 = \frac{1}{L^3}(2x^3 - 3x^2L + L^3)$ $N_2 = 1/L^3 (x^3L - 2x^2L^2 + xL^3]$ $N_3 = \frac{1}{L^3}(-2x^3 + 3x^2L)$ $N_4 = \frac{1}{L^3}(x^3L - x^2L^2).$	5M.
b)	 Divide the beam into two elements.	

QNO	ANSWER	MARKS
	 Calculate slopes, vertical forces. Calculate stiffness equations for Element ① $K = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}.$ $\begin{bmatrix} v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} F_{1y} \\ M_1 \\ F_{2y} \\ M_2 \end{bmatrix}$ Element ② Assemblem stiffness matrix of ① & ② Apply B.C's. Calc. Displacement, slopes.	5M.

QNO	ANSWER	MARKS
5A)	 $E = 2\times10^5$ N/mm^2 $A_2 = A_1 = 1000$ mm^2 $l_2 = 50$mm $l_1 = 10$ mm. Element ① $l_{e_1} = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$ $l_1 = \frac{x_2-x_1}{l_{e_1}}$ $m_1 = \frac{y_2-y_1}{l_{e_2}}$ Element ② $l_{e_2} = \sqrt{(x_3-x_2)^2 + (y_3-y_2)^2}$ $l_2 = \frac{x_3-x_2}{l_{e_2}}$, $m_2 = \frac{y_3-y_2}{l_{e_2}}$	

QNO	ANSWER	MARKS
	Calculate Assembled Stiffness. $K_1 + K_2 \Rightarrow k_1 = \frac{A_1E_1}{l_1}\begin{bmatrix} l_1^2 & l_1m_1 & -l_1^2 & -l_1m_1 \\ l_1m_1 & m_1^2 & -l_1m_1 & -m_1^2 \\ -l_1^2 & -l_1m_1 & l_1^2 & l_1m_1 \\ -l_1m_1 & -m_1^2 & l_1m_1 & m_1^2 \end{bmatrix}$ $k_1 = \frac{1000\times 2\times 10^5}{le_1}\begin{bmatrix} \text{〃} & \text{〃} \end{bmatrix}$ Applying BC's. Calculate Displacement and stresses. $\sigma_1 = \frac{E}{le_1}\begin{bmatrix} -l_1 & -m_1 & l_1 & m_1 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix}$ $\sigma_2 = \frac{E}{le_1}\begin{bmatrix} -l_2 & -m_2 & l_2 & m_2 \end{bmatrix}\begin{bmatrix} u_5 \\ u_4 \\ u_5 \\ u_6 \end{bmatrix}$	10M

QNO	ANSWER	MARKS
6A)	B) $\sigma \propto \epsilon$. $\sigma = E \cdot e$. $e = \frac{\sigma}{E}$. Stress in x-direction produces a positive strain in x-direction $\Rightarrow e'_x = \frac{\sigma_1}{E}$ Stress in y-direction produces -ve strain in x-direction $e''_x = -\frac{\nu \sigma_y}{E}$ ν - poisson's Ratio. Similarly in z-direction $-\ e'''_x = \frac{-\nu \sigma_z}{E}$ $e_x = \frac{\sigma_x}{E} - \frac{\nu \sigma_y}{E} - \frac{\nu \sigma_z}{E}$ $e_y = -\nu \frac{\sigma_x}{E} + \frac{\sigma_y}{E} - \frac{\nu \sigma_z}{E}$.	5M.

QNO	ANSWER	MARKS

$$e_z = -\nu \frac{\sigma_x}{E} - \nu \frac{\sigma_y}{E} + \frac{\sigma_z}{E}$$

solving above strain eq's normal stresses will be obtained.

$$\rightarrow \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} (1-\nu) & \nu & \nu & 0 & 0 & 0 \\ \nu & (1-\nu) & \nu & 0 & 0 & 0 \\ \nu & \nu & (1-\nu) & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1-2\nu}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1-2\nu}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix} \begin{bmatrix} e_x \\ e_y \\ e_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{bmatrix}$$

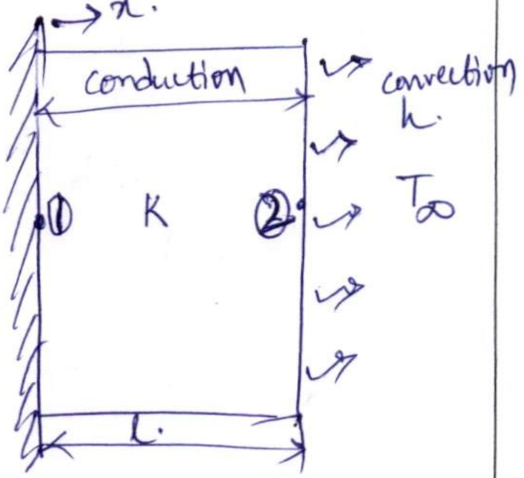
QNO	ANSWER	MARKS
7A)	Stress = D·B·u.	

$$= \begin{bmatrix} \sigma_r \\ \sigma_\theta \\ \sigma_z \\ \tau_{rz} \end{bmatrix} = D \cdot B \cdot \begin{bmatrix} u_1 \\ w_1 \\ u_2 \\ w_2 \\ u_3 \\ w_3 \end{bmatrix}$$

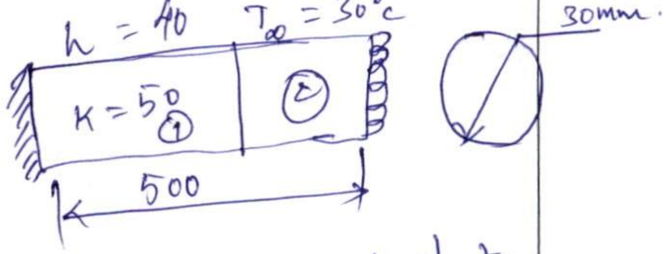
$$D = \frac{E}{(1+v)(1-2v)} \begin{bmatrix} 1-v & v & v & 0 \\ v & 1-v & v & 0 \\ v & v & 1-v & 0 \\ 0 & 0 & 0 & \frac{1-2v}{2} \end{bmatrix}$$

$$B = \frac{1}{2A} \begin{bmatrix} \beta_1 & 0 & \beta_2 & 0 & - \\ \frac{\alpha_1}{r} + \beta_1 + \frac{\gamma_1 z}{r} & 0 & \frac{\frac{\alpha_2}{\cancel{\gamma}} + \beta_2 + \gamma_2 z}{r} & 0 & - \\ 0 & \gamma_1 & 0 & \gamma_2 & - \\ \gamma_1 & \beta_1 & \gamma_2 & \beta_2 & - \end{bmatrix}$$

QNO	ANSWER	MARKS
	$\alpha_1 = r_2 z_3 - r_3 z_2$ $\alpha_2 = r_3 z_1 - r_1 z_3$ $\alpha_3 = r_1 z_2 - r_2 z_1$ $\beta_1 = z_2 - z_3$ $\beta_2 = z_3 - z_1$ $\beta_3 = z_1 - z_2$ $r_1 = r_3 - r_2$ $r_2 = r_1 - r_2$ $r_3 = r_2 - r_1$ Substitute (α_1, β_1, r_1) $(\alpha_2 \beta_2 r_2)$ $(\alpha_3 \beta_3 r_3)$ calculate various stresses. —— 2M	8M

QNO	ANSWER	MARKS
8 A)	b) Consider 1D element  $K_c = \frac{AK}{l}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$ $N = [N_1 \quad N_2] = \left[\frac{l-x}{l} \quad \frac{x}{l}\right]$ when $x = l$. $N = [0 \quad 1]$, $N^T = \begin{bmatrix}0\\ 1\end{bmatrix}$ $[K_h]_{end} = h\begin{bmatrix}0 & 1\\ 0 & 1\end{bmatrix} . A.$ $K = K_c + K_h$ $= \frac{Al}{l}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} + hA\begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix}$	

QNO	ANSWER	MARKS
	$\left[F_h\right]_{end} = h \cdot T_{\infty} \cdot A \cdot$ $\left[F_h\right]_{end} = h \cdot T_{\infty} \cdot A \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $F = K \cdot T \cdot$ $\Rightarrow h \cdot T_{\infty} \cdot A \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{AK}{l} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + hA \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} T_1 \\ T_2 \end{bmatrix}$ This is F·E equation for 1D slab.	5M.

QNO	ANSWER	MARKS
9A)	Discritere the pin into two elements  $q = 400 \, kw \mid m^2 K$. F.E is for ~~Node~~ element 1. $\frac{AK}{L_1}\begin{bmatrix}1 & -1 \\ -1 & 1\end{bmatrix} + \frac{hPl_1}{6}\begin{bmatrix}2 & 1 \\ 1 & 2\end{bmatrix}\begin{bmatrix}T_1 \\ T_2\end{bmatrix} =$ $\frac{QAL_1 + Ph_\infty T l_1}{2}\begin{bmatrix}1 \\ 1\end{bmatrix}$ $P =$ ~~2~~($\pi d = 2 \times 30 = 60mm$. $A = \pi r^2 = \pi (30)^2$. Calculate F.E for ~~Node~~ Element 2.	5M
	Assemble the Finite Element equation for Element ① and ② Calculate temp. distribution values.	5M

QNO	ANSWER	MARKS
10A)	For bar Element: $N_1 = 1-\frac{x}{l}$, $N_2 = \frac{x}{l}$. $m = \int_V \rho N^T \cdot N \cdot dv$ $= \rho A \int_0^l \begin{bmatrix} N_1 \\ N_2 \end{bmatrix} \cdot [N_1, N_2] \, dx$. $= \rho A \int_0^l \begin{bmatrix} (1-\frac{x}{l})^2 & \frac{x}{l}-\frac{x^2}{l^2} \\ \frac{x}{l}-\frac{x^2}{l^2} & \frac{x^2}{l^2} \end{bmatrix} dx$. $= \rho A \begin{bmatrix} l/3 & l/2 - l/3 \\ l/2 - l/3 & l/3 \end{bmatrix}$ $= \frac{\rho A l}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$. For Beam Element:	5m.

QNO	ANSWER	MARKS
	$N_1 = 1 - \frac{3x^2}{l^2} + \frac{2x^3}{l^3}$ $N_2 = x - \frac{2x^2}{l} + \frac{x^3}{l^2}$ $N_3 = \frac{3x^2}{l^2} - \frac{2x^3}{l^3}$ $N_4 = \frac{-x^2}{l} + \frac{x^3}{l^2}$ $m = \int_V \rho N^T \cdot N \cdot dV.$ $= \rho A \int_0^l \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \end{bmatrix} \begin{bmatrix} N_1 & N_2 & N_3 & N_4 \end{bmatrix} dx.$ Substitute N_1, N_2, N_3, N_4 values and perform integration. , the beam element	

QNO	ANSWER	MARKS

Mass matrix as

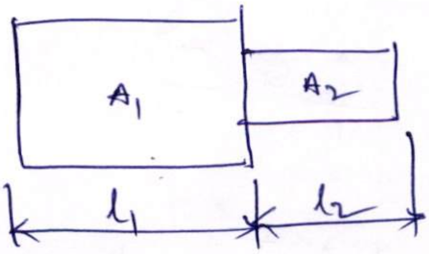
$$m = \frac{\rho A l}{420}\begin{bmatrix}156 & 22l & 54 & -13l\\ 22l & 4l^2 & 13l & -3l^2\\ 54 & 13l & 156 & -22l\\ -13l & -3l^2 & -22l & 4l^2\end{bmatrix}$$

11A) Natural frequency for unconstrained bar

$A_1 = 1000 mm^2$

$A_2 = 500 mm^2$

$l_2 = L_1 = 150 mm$



$E = 2\times10^5 N/mm^2$ } Assumption.

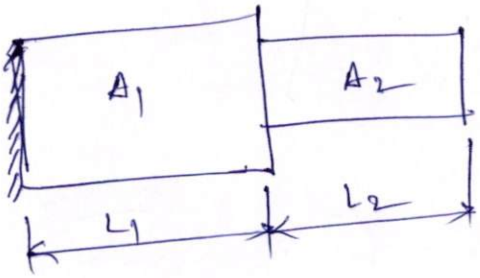
$\rho = 7800 kg/m^3$

$$K_1 = \frac{A_1E_1}{L_1}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$$

Calculate for Element ①

$$m_1 = \frac{\rho_1 A_1 l_1}{6}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}.$$

QNO	ANSWER	MARKS
	Calculate for Element ② $K_2 = \frac{A_2 E_2}{l_2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ $m_2 = \frac{\rho_2 A_2 l_2}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ Assemble the stiffness and massmatrix for Element ① & ② and substitute in dynamic eq. $[K - \omega^2 \cdot m] \cdot u = 0$. Applying B.C's and calculate "λ" values. from λ calculate ω values.	5M

QNO	ANSWER	MARKS
	Natural frequency for constrained Bar  calculate K_1, M_1 (Stiffnes, mass matrix) for Element ①, Calculate K_2, M_2 (stiffnes, mass matrix) for Element ②. Assemble the stiffness and mass matrix for Element ① and ② Apply B.C's.	-

QNO	ANSWER	MARKS
	Substitute the Assembled stiffness and massmatrix values in dynamic Equation $[K - \omega^2 \cdot m] \cdot u = 0.$ Apply B.C's $u_1 = 0.$ calculate λ values and ω values. where λ - Eigen values ω - natural frequency.	5M.

QNO	ANSWER	MARKS